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HW Six , Math 530, Fall 2014

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QUESTION 1. Let $(G, *)$ be a group.

(i) Let G, H be finite group and $L = G \times H$. We know that in general that if M is a subgroup of L, then M need not be of the form $B \times A$ for some subgroup B of G and some subgroup A of H. However, if L is cyclic, then prove that every subgroup of L is of the form $K \times V$ for some subgroup K of G and some subgroup V of H.

(ii) Let F be be a group of order p^2 for some prime integer p and assume that F is not cyclic. Prove that F is group-isomorphic to $(Z_p, +) \times (Z_p, +)$.

(iii) Let F be an abelian group of order p^2q for some prime integers p, q. Assume that F is not cyclic. Prove that F is group-isomorphic to $(Z_p, +) \times (Z_p, +) \times (Z_q, +)$.

(iv) Let F be an abelian group such that $|F| = n = mp^k$ where p is prime and $gcd(p, m) = 1$ [Hint: Use induction: Assume that every abelian group of order $dp^c < n$ where $gcd(p, d) = 1$ has a subgroup of order p^c]

(v) Prove that every abelian group of order pq for some prime integers p, q is cyclic.

(vi) Let G is a finite group and H be a subgroup of G of order m such that H is not normal subgroup of G. Prove that G has at least $|G|/|N_G(H)|$ distinct subgroups of order m.(Hint: Let $a * N_G(H)$, $b * N_G(H)$ be distinct left cosets of $N_G(H)$. Prove that aHa^{-1} and bHb^{-1} are distinct)

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